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NATURE OF CONFORMALLY COUPLED SCALAR FIELD IN COSMOLOGICAL MODELS

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ABSTRACT

The cosmos appears to be expanding, according to recent measurements of supernovae and the cosmic microwave background. Gravitational scalar tensor theories produce cosmic models that logically incorporate a late-time fast expansion. In a different hypothesis, the stretch of the universe is produced using a scalar field (quintessence), much like the early inflation. In this study, we focus on a particular category of scalar field cosmological models. It has a quartic potential and a conformally connected scalar field. The cosmic dynamics are detailed, and the braking scale is assessed in the model. For the values of the scale, that are supplied, the expansion accelerates in the late stages.

***KEYWORDS*:** *Cosmological Model, Deceleration, Scalar Field Parameter*

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INTRODUCTION

The largely accepted theory of the nature of the universe's structure and evolution has been re-examined in light of recent evidence of the cosmos' accelerated expansion (Copeland et al., 2006; Perlmutter et al., 1999; Garnavich et al., 1998). The cosmological constant is the most likely choice to explain the acceleration of space-time (Weinberg, 1989). Consider the negative pressure that occurs from a non-zero vacuum energy as a counterpart to this (Linde, 1982; Starobinski, 1982; Guth, 1981). The idea that the universe is composed of a unique material termed quintessence (Steinhardt and Caldwell, 1998) offers an alternate explanation for the observed acceleration. Intriguingly, the presence of quintessence has an impact on Logunov and colleagues' relativistic theory of gravitation (RTG): rather than expanding more quickly, the universe slows down, stops, and then contracts to a scale factor minimum before beginning a new cycle of expansion (Chernin, 2008; Gershtein et al., 2003). It should be emphasized that before, mostly for philosophical reasons, the oscillating nature of the universe's evolution was initially posited (Markov and Aman, 1984). The transit through the cosmic singularity and the rise in entropy from cycle to cycle cause the oscillation mode of the Friedmann closed model to be broken up (Tolmen, 1949).

In this study, we examine a class of scalar field cosmological models. The paper is structured as follows. The conformally linked scalar field cosmology model with the barotropic equation of state, non-gravitational matter, and the Higgs potential is discussed in the next section. The universe's accelerated expansion models are listed. The key findings of the paper are outlined in Section 3.

Cosmological Matter with a Scalar Field (Conformally Coupled)

The quartic potential and the conformally connected scalar field interact (Stanukovich and Melnikov, 1983; Melnikov, 2011; Bronshtein and Semendjaev, 1980) as shown below:

$$W = \int\left[-\left(\frac{1}{2k}-\frac{\psi^2}{12}\right)R+\frac{1}{2}(\nabla\psi)^2-\frac{\lambda}{12}\psi^4+L_m\right]\sqrt{-g}d^4x. \tag{2.1}$$

The following field equations are obtained as:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=k\left(1-\frac{k\psi^2}{6}\right)^{-1}\left[\tau_{\mu\nu}+T_{\mu\nu}^{mat}\right] \tag{2.2}$$

$$\nabla^2\psi-\frac{R}{6}\psi+\frac{\lambda}{3}\psi^3=0, \tag{2.3}$$

where

$$\tau_{\mu\nu}=\frac{2}{3}\nabla_\mu\psi\nabla_\nu\psi-\frac{1}{6}g_{\mu\nu}(\nabla\psi)^2-\frac{\psi}{3}\nabla_\mu\nabla_\nu\psi+\frac{\psi}{3}g_{\mu\nu}\nabla^2\psi+\frac{\lambda}{12}\psi^4 g_{\mu\nu},\ T_{\mu\nu}^{mat}=(\varepsilon+P)U_\mu U_\nu-Pg_{\mu\nu}, \tag{2.4}$$

In this instance, the Eq. (2.2) is condensed by taking (2.3) into consideration

$$-R = kT = k(\varepsilon-3P) \tag{2.5}$$

The homogeneous and isotropic universe model is often thought to work well with geometry using the Friedmann-Robertson-Walker metric. The aforementioned equations consequently hold true for the flat model of matter having the equation of state $P = \alpha\varepsilon$

$$\frac{\dot{R}^2}{R^2}=\frac{k}{3(1-k\psi^2/6)}\left[\frac{\dot{\psi}^2}{2}-\psi\dot{\psi}\frac{\dot{R}}{R}+\varepsilon\right], \tag{2.6}$$

$$-\left(\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)=\frac{k}{3(1-k\psi^2/6}\left[\frac{\dot{\psi}^2}{2}-2\frac{\dot{R}}{R}\psi\dot{\psi}-\psi\ddot{\psi}+3P\right], \tag{2.7}$$

$$\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=\frac{k}{6}(\varepsilon-3P), \text{ (2.8)}$$

$$\ddot{\psi}+3\frac{\dot{R}}{R}\dot{\psi}+\psi\left(\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)+\frac{\lambda}{3}\psi^3=0 \text{ (2.9)}$$

$$\dot{\varepsilon}=-\frac{3\dot{R}}{R}(\varepsilon+P), \varepsilon=\varepsilon_0/R^{3(1+\alpha)} \text{ (2.10)}$$

Here the scale factor is R(t). One of the system's equations, (2.6) - (2.10), is derived from the others.

The following expression for the deceleration parameter is obtained from (2.8)

$$q = 1 - \frac{k}{6H^2}(\varepsilon - 3P), \tag{2.11}$$

Here the Hubble parameter is $H = \dot{R}/R$, because it follows from the requirement $q \leq 0$ that there is a chance that the cosmos is expanding uniformly or quickly is

$$\varepsilon \geq 2.\frac{3H^2}{8\pi G} + 3P. \tag{2.12}$$

(Let us consider that the framework of General Relativity one has $\varepsilon_{crit} = 3H^2/(8\pi G)$) The problem q is phrased as follows for the variation of α, a feature of cosmology:

$$q = 1 - \frac{k\varepsilon}{6H^2}, \text{matter dominated } (\alpha = 0),$$

$$q = 1 - \frac{2k\varepsilon}{3H^2}, \text{vacuum model } (\alpha = -1),$$

$$q = 1, \text{ radiation dominated } (\alpha = 1/3), \tag{2.13}$$

$$q = 1 + \frac{k\varepsilon}{3H^2}, \text{stiff fluid } (\alpha = 1).$$

$q = 1$, scalar field conquered ($\varepsilon = 0$, $P = 0$),

The Hubble parameter can be stated using (2.8) in the following way:

$$H^2 = \frac{k\varepsilon_0}{3R^{3(1+\alpha)}} + \frac{\beta}{R^4},$$

β is an integration constant.

Let's examine each unique instance in turn:

(i) $R = \sqrt{2ct + b}$, $H = c/R^2$results from the scalar curvature going to zero for the scalar field rules (ε, $P = 0$) or when it comes to the equation of state $P = \varepsilon/3$

$\psi = c_2 - c_1/(c\,R)$ with

$$c^2\left(3 - C_2^2 k/2\right) = 0 \Rightarrow C_2^2 = 6/k, \psi = \sqrt{6/k} - c_1/cR, \tag{2.14}$$

i.e. for adequately large R we have $\psi \rightarrow \sqrt{6/k}$.

(ii) The systematic structure for the functions R,ψ, H be unchanged, if the radiation is dominating ($\alpha = 1/3$) while the scalar field is present, and equation (2.14) represents

$$c^2\left(3 - c_2^2 k/2\right) = k\varepsilon_0, \tag{2.15}$$

so that $c_2 = \sqrt{6/k - 2\varepsilon_0/c^2}$. In both scenarios (i) and (ii), the universe's growth is continuously slowed down, and as it gets older, scalar fields often have constant values.

(iii) $P = 0$, $\varepsilon = \varepsilon_0//R^3$, which means that we get the Hubble parameter for the matter-dominant era,

$$H^2 = \frac{k\varepsilon}{3} + \frac{\beta}{R^4} = \frac{k\varepsilon_0}{3R^3} + \frac{\beta}{R^4} \tag{2.16}$$

and, correspondingly,

$$q = 1 - \frac{k\varepsilon_0 R / 6}{k\varepsilon_0 R / 3 + \beta} \tag{2.17}$$

The first component on the RHS of (2.16), which dominates near the end of the cosmic expansion ($R\to\infty$), results in the standard matter-dominated development and a decelerated expansion. One has $q \to ½$ in this limit (as in GR).

For $\beta > 0$, we have the following solution

It directly integrates Eq. (2.16). The solution is given in the form of

$$R = \frac{3\beta}{k\varepsilon_0} v, t / t_2 = (v-2)\sqrt{v+1} + 2, 0 \le \upsilon \le \infty, \tag{2.18}$$

in which the notation $t_2 = 6|\beta|^{3/2}/(k\varepsilon_0)^2$is used. At the beginning and ending of the cosmic growth, we have, respectively, $R(t) \propto t^{1/2}$at $t \to 0$, and $R(t) \propto t^{2/3}$ at $t \to\infty$. The expansion always slows down in this circumstance.

For $\beta < 0$,

$$R = \frac{3|\beta|}{k\varepsilon_0} v, t / t_2 = (v+2)\sqrt{v-11}\ , 0 \le \upsilon \le \infty. \tag{2.19}$$

Currently, the early-time dynamics are completely different, while the late-time evolution matches that of the earlier scenario. An asymptotic behaviour of $R \approx R_{min} + (k\varepsilon_0)^3\ t^2 / (4\beta^2)$, $t \to 0$, and $R_{min} = 3|\beta| / (k\varepsilon_0)$ are the initial conditions for the expansion. Early on, the expansion is accelerated and the inequality $1 \le v \le 2$replaces the prerequisite $q \le 0$.

It is easy to find the scalar field's temporal evolution in the case where $\lambda = 0$

$\ddot{U} + \dot{U}\dot{R} / R = 0$, (2.20)

with the representation $U = R\psi$.

For the scalar field (2.16) from here, we find

$$\psi = \text{const}\begin{cases}\sqrt{v+1} / r, 0 \le v < \infty & \text{for}\beta > 0.\\ \sqrt{v-1} / r, 1 \le v < \infty & \text{for}\beta < 0.\end{cases} \tag{2.21}$$

Where the scale factor ψ and v are related by the formulas (2.18 and (2.19) At the end of the expansion, one has $\psi\to 0$ in both occurrences of the sign for one. For non-zero values of the parameter, the dynamics of the field at the beginning of the expansion are fundamentally different: for positive values, we obtain, $\psi\to\infty$, $1 \to 0$, whereas the field tends to zero for negative values, $\psi\to 0$, $1 \to 0$.

(iv) We have a set of equations for the equation for the condition $P = \varepsilon$

$$\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = \frac{2k\varepsilon}{3}. \tag{2.22}$$

$$\ddot{\psi} + 3\dot{\psi}\sqrt{\frac{k\varepsilon_0}{3} + \frac{\beta}{R^4}} + \frac{2k\varepsilon_0}{3}\psi = 0 \tag{2.23}$$

Substituting $v = R^2$ equation (2.23) becomes

$$i^2 - \frac{4k\varepsilon}{3} v^2 = \frac{\beta}{4} \tag{2.24}$$

It becomes

$$R(t) = R_0 \sinh^{1/2} v,\ v = 2t\sqrt{k\varepsilon/3} \tag{2.26}$$

For $\beta > 0$

$R(t) = R_0 \cosh^{1/2} v$(2.26)

For $\beta < 0$, where $4R_0^2 = \sqrt{3|\beta|/(k\varepsilon}.$, equation (2.23) is reduced to the equation

$$\psi''(\tau) + \frac{3}{2} f(\tau)\psi'(\tau) + \frac{1}{2}\psi = 0 \quad (2.27)$$

where f = thanτ for $\beta < 0$ and f = coshτ for $\beta > 0$. The hyper geometric function is used to express the answer to equation (2.27). The Eq. (2.27) is reduced to when the scale factor is big, which corresponds to the later phases of the cosmic growth, $t \to \infty$, one has $R \propto e\tau/2$,

$$\ddot{\psi} + 3\sqrt{\frac{k\varepsilon}{3}}\dot{\psi} + \frac{2k\varepsilon}{3}\psi = 0,$$

and as a result, for ψ one finds

$$\psi \approx \mathrm{C1e}^{-t} + \mathrm{C2e}^{-1/2},$$

Here C_1, $C_2$are constants. The scalar field vanishes at this limit, one has $q \approx -1$.

CONCLUSION

Scalar fields are essential in hypothetical models that show the growth of the cosmos. Now, we have examined the cosmic dynamics of a specific class of scalar field models. We have demonstrated, by means of this model that the cosmos enters the phase of accelerated expansion when the dynamic term of the scalar field prevails during the phase of decelerating expansion. When the scalar field reaches a constant value in the latter stages, we see an exponential expansion. There is a description of the conditions that lead to an accelerated period of cosmic expansion. We have particularly shown that the expansion is always slowed down in the presence of radiation-type materials or a dominant scalar field. The collection of dust is real in this model. The cosmos starts to grow at in (2.16), starting from a determinate worth of the scale issue, initially expanding more swiftly before slowing down and ultimately arriving at a location where the scalar field tests to zero. The early universe mostly relies on the value of the incorporation constant 'b' on the RHS of the cosmological equations when the extra source is provided by the cosmological constant. The analogous scaling factor, which has a smallest non-zero value for negative 'b', is given by formulas (2.25) and (2.26). The scalar field diminishes exponentially at the end of the cosmic expansion, and the expansion is now driven exponentially by the cosmological constant.

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